Calculator Lab: Numerical Integration II

In this lab we will examine the errors in numerical approximations of the integrals

$$\int_0^1 3x^2 dx \qquad \text{and} \qquad \int_1^9 \frac{3\sqrt{x}}{2} dx.$$

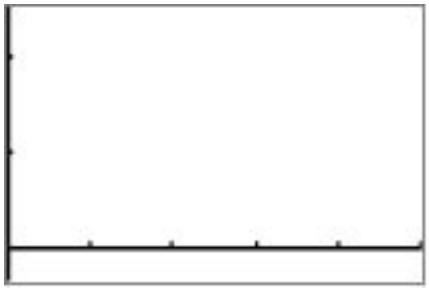
Part I: Analytical Approach

1. (a) Use the Fundamental Theorem of Calculus to evaluate $\int_0^1 3x^2 dx$. (In other words, find an antiderivative of $3x^2$ and use that antiderivative to evaluate the integral.) Note that this value is *the actual value of the integral.*

   (b) Use the Fundamental Theorem of Calculus to evaluate $\int_1^9 \frac{3}{2}\sqrt{x} dx$.

Part II: Numerical Approach

2. (a) Sketch a graph of $3x^2$ with `xMin` $= 0$, `xMax` $= 1$ `yMin` $= 0$, `yMax` $= 3.5$ in the box on the left. Sketch a graph of $\frac{3\sqrt{x}}{2}$ with `xMin` $= 0$, `xMax` $= 5$ `yMin` $=$ -0.5, `yMax` $=$ 4 in the box on the right.



(b) Will LEFT(n) be an upper or lower bound for $\int_0^1 3x^2 dx$? For $\int_1^9 \frac{3}{2}\sqrt{x}dx$?

(c) Will RIGHT(n) be an upper or lower bound for $\int_0^1 3x^2 dx$? For $\int_1^9 \frac{3}{2}\sqrt{x}dx$?

(d) Will MID(n) be an upper or lower bound for $\int_0^1 3x^2 dx$? For $\int_1^9 \frac{3}{2}\sqrt{x}dx$?

(e) Will TRAP(n) be an upper or lower bound for $\int_0^1 3x^2 dx$? For $\int_1^9 \frac{3}{2}\sqrt{x}dx$?

3. (a) Use the `ALLSUMS` program to fill in the following table:

Approximations of $\int_0^1 3x^2 dx$.

n	LEFT(n)	RIGHT(n)	MID(n)	TRAP(n)
5				
10				
20				
50				

(b) Use the table above and your solution to #1(a) to compute the following table:

Error = Numerical Approximation - $\int_0^1 3x^2 dx$.

n	LEFT(n) - $\int_0^1 3x^2 dx$	RIGHT(n) - $\int_0^1 3x^2 dx$	MID(n) - $\int_0^1 3x^2 dx$	TRAP(n) - $\int_0^1 3x^2 dx$
5				
10				
20				
50				

(c) What happens to the error term for LEFT(n) (and RIGHT(n)) when n is doubled? When n is multiplied by 5? When n is multiplied by 10?

(d) What happens to the error term for MID(n) (and TRAP(n)) when n is doubled? When n is multiplied by 5? When n is multiplied by 10?

4. (a) Use the `AllSUMS` program to fill in the following table:

$$\text{Approximations of } \int_1^9 \frac{3\sqrt{x}}{2}dx.$$

n	LEFT(n)	RIGHT(n)	MID(n)	TRAP(n)
5				
10				
20				
50				

(b) Use the table above and your solution to #1(b) to compute the following table:

$$\text{Error} = \text{Numerical Approximation - } \int_1^9 \frac{3\sqrt{x}}{2}dx.$$

n	LEFT(n) - $\int_1^9 \frac{3\sqrt{x}}{2}dx$	RIGHT(n) - $\int_1^9 \frac{3\sqrt{x}}{2}dx$	MID(n) - $\int_1^9 \frac{3\sqrt{x}}{2}dx$	TRAP(n) - $\int_1^9 \frac{3\sqrt{x}}{2}dx$
5				
10				
20				
50				

(c) What happens to the error term for LEFT(n) (and RIGHT(n)) when n is doubled? When n is multiplied by 5? When n is multiplied by 10?

(d) What happens to the error term for MID(n) (and TRAP(n)) when n is doubled? When n is multiplied by 5? When n is multiplied by 10?

Part III: Wrap-up

5. (a) In general, what do you expect to happen to the error term for LEFT(n) (and RIGHT(n)) when n is multiplied by k?

   (b) In general, what do you expect to happen to the error term for MID(n) (and TRAP(n)) when n is multiplied by k?

6. (a) What else do you notice about the errors for MID(n) versus the errors for TRAP(n)?

   (b) From your answer above, can you obtain a better numerical approximation method?